Optimal Management of Systems with Thresholds

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1. Fishery management with no threshold. This shows the role of the discount rate in determining optimal strategies.

2. Fishery with a threshold: here the harvest target depends upon stock size as well as the discount rate.

3. Optimal phosphorus loading in a lake, with a threshold for recycling of P from sediments.

4. Discounting schemes and their effects on optimal strategies.

Management of a fishery

In year t, there are R_t individuals (recruits). Harvest H_t, leaving stock $S_t = R_t - H_t$ to reproduce.

The number of recruits in the next year is

$$R_{t+1} = F(S_t). \tag{1}$$

Present Value starting at a size R at time t is given by

$$V_t(R) = \frac{1}{w_t} \sum_{\tau=t}^{\infty} w_\tau H_\tau = \frac{1}{w_t} \sum_{\tau=t}^{\infty} w_\tau \left[F(S_{\tau-1}) - S_\tau\right], \tag{2}$$

where w_τ is a system of weights.

An interior maximum of the right–hand side of (2) (denoted by S_t^*) must satisfy

$$F'(S_t^*) = \frac{w_t,}{w_{t+1}}, \quad \text{and} \quad 0 < S_t^* < R_t. \tag{3}$$

Assuming that a single such interior maximum exists, then the optimal choice of H_t is

$$H_t = \begin{cases} 0 & \text{if} \quad R_t \leq S_t^*, \\ R_t - S_t^* & \text{if} \quad R_t > S_t^*. \end{cases} \tag{4}$$

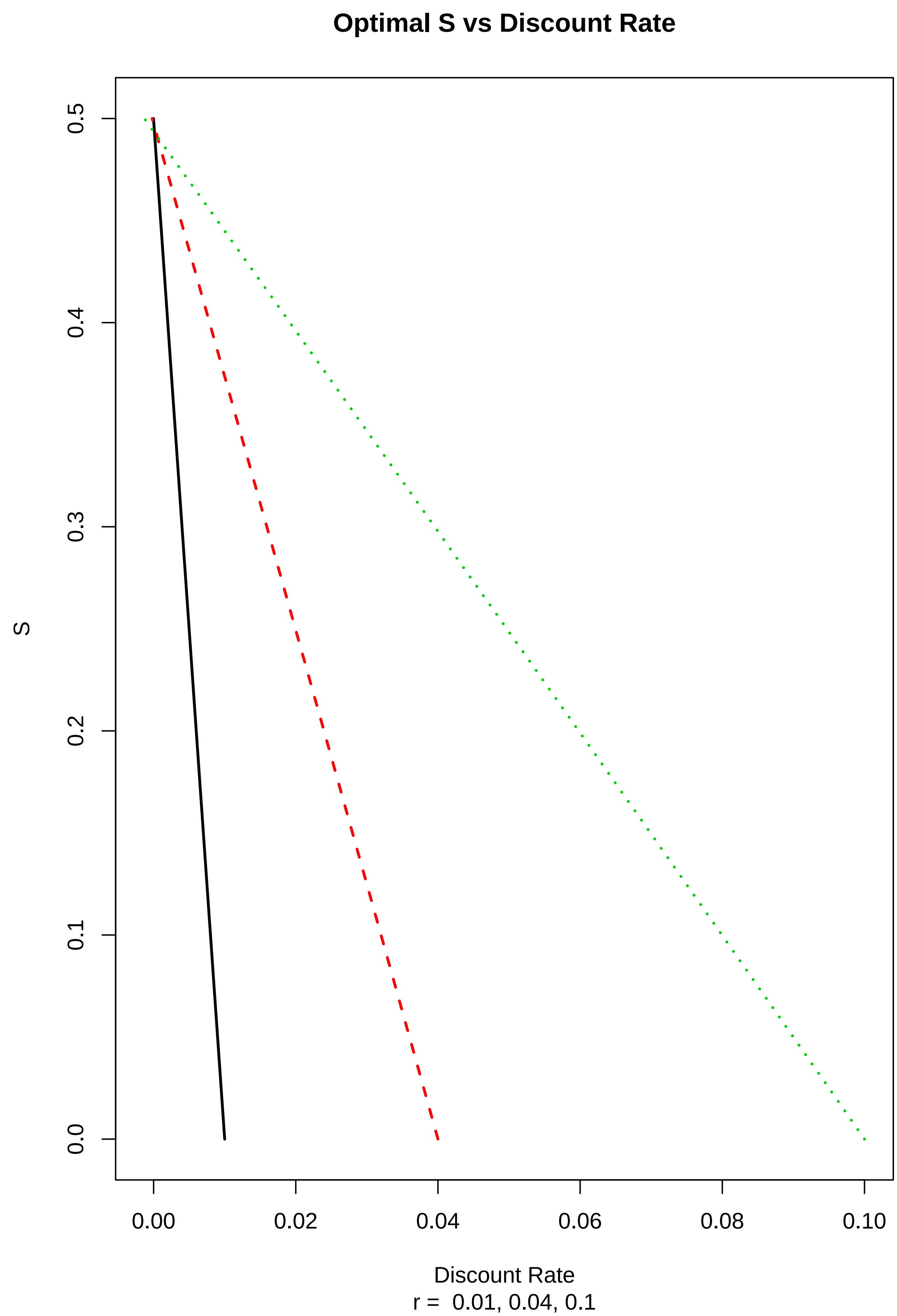
If $S^* \leq 0$, then the optimal strategy is to harvest the entire stock.

Assume the Ricker form for stock–recruit relationship:

$$F(S) = Se^{r(1-S)}, \tag{5}$$

where r is a growth parameter.

We can plot S^* vs $\delta = \log(\frac{w_t,}{w_{t+1}})$ for various values of r.


Optimal S vs Discount Rate
0.5
0.4
0.3
0.2
0.1
0.0
S
0.00
0.02
0.04
0.06
0.08
0.10
Discount Rate
r = 0.01, 0.04, 0.1

Random dynamics and thresholds

Fish stocks are notorious for variability of recruitment; it was once thought that recruitment was independent of stock size!

The stock may fail to replace itself if the number of spawners is too low (threshold).

This may be due to predators whose numbers are limited, but which may have severe effects on small cohorts.

Both of these effects lead to higher optimal stock sizes, (precautionary policies), but the optimal policy calls for extinction if the stock falls below a certain size, related to the threshold.

In general, if errors in direction have more serious consequences than errors in the other, then precaution is indicated.

Management of a lake

We optimize the expected discounted net benefits of phosphorus (P) loadings for a potentially eutrophic lake.

The benefits accrue to agricultural interests from activities that result in loading, while costs accrue to other interests from the resulting deterioration of water quality.

We extend earlier results in Carpenter, Ludwig and Brock (1999) to account for dependence of P recycling upon the concentration of P in sediments.

This system exhibits multiple equilibria, a threshold, and several temporal scales.

Our analysis focuses on two state variables: phosphorus in the water (P) and phosphorus in the lake sediment (M).

The single control variable is phosphorus loading (L).

We assume that utility is given by

$$U(L,P) = \alpha L - \beta_1 P - \beta_2 P^2, \tag{6}$$

where L is the phosphorus loading, α is positive and β_i are nonnegative.

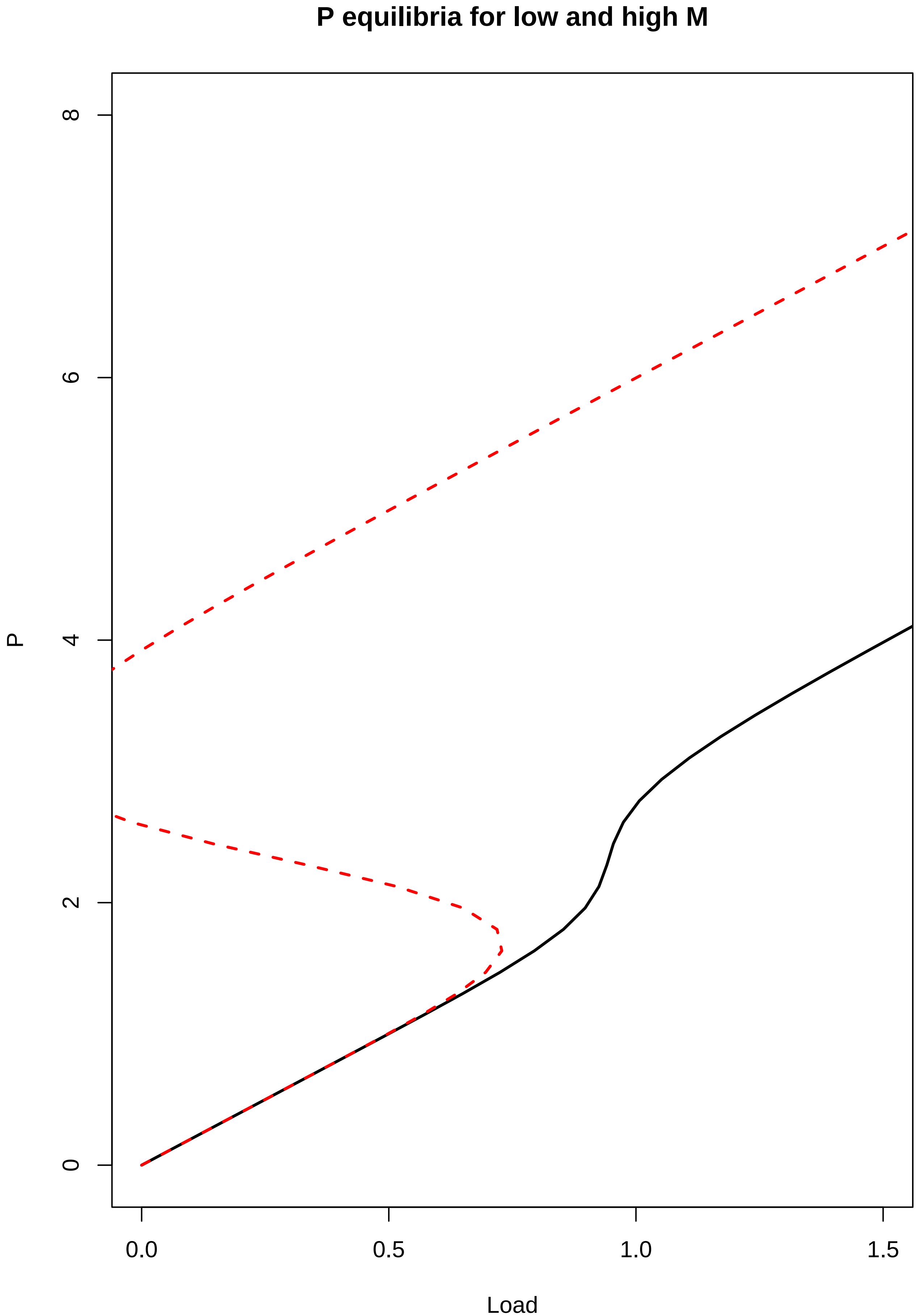
Phosphorus dynamics:

$$P_{t+1} = P_t(1 - s - h) + L_t N_t + rM_t F(P_t), \tag{7}$$

where s is the sedimentation rate, h is the hydrological flushing rate, N_t is lognormal, r is the recycling rate, and

$$F(P) = \frac{P^8}{P_c^8 + P^8}. \tag{8}$$

Here P_c is a critical level of P.


P equilibria for low and high M
P
Load
0
2
4
6
8
0.0
0.5
1.0
1.5

Mud dynamics:

$$M_{t+1} = M_t(1-b) + sP_t - rM_tF(P_t). \tag{9}$$

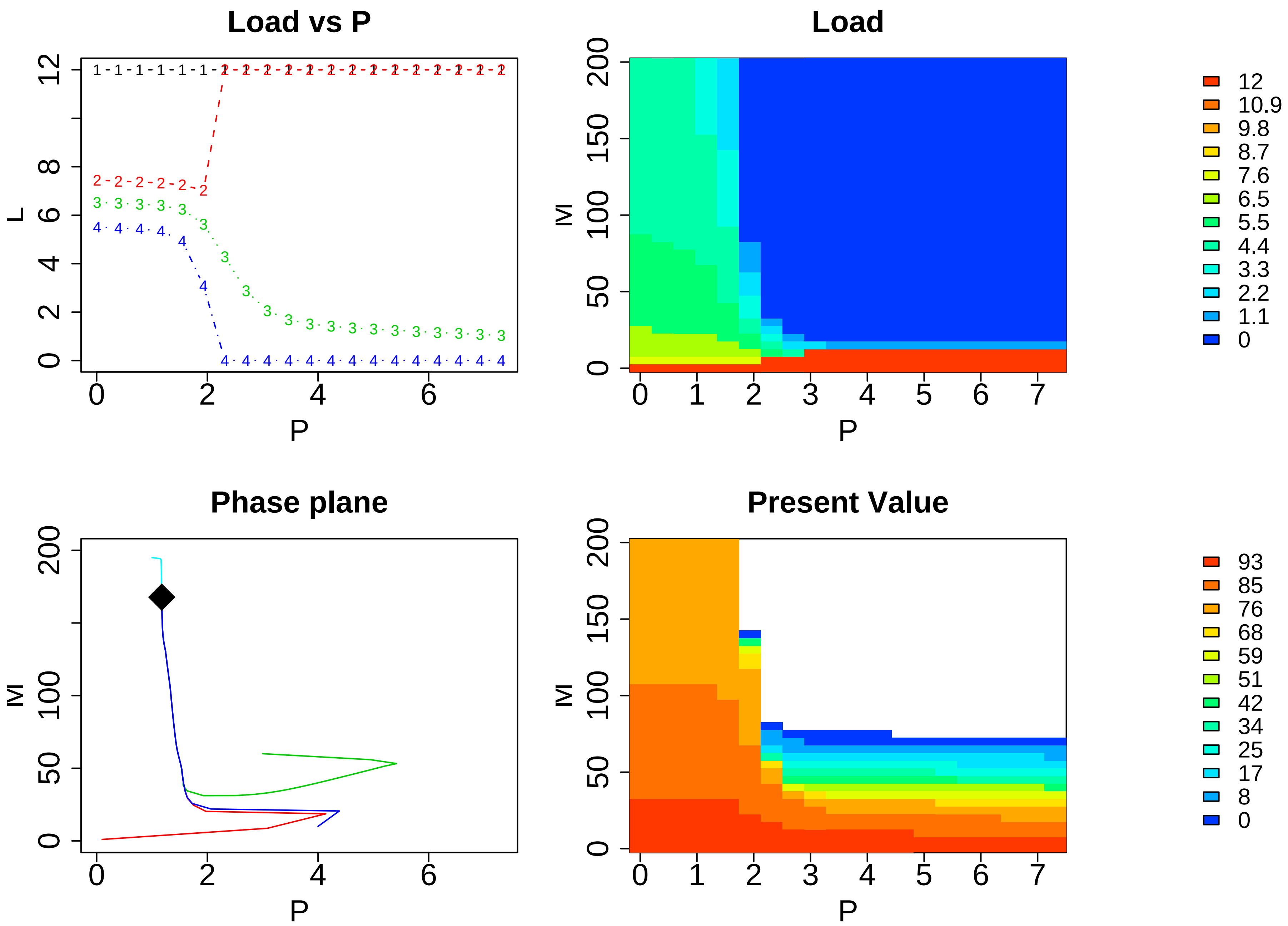
We compute the optimal loading policy by solving the dynamic programming equation

$$V_t(P_t, M_t) = U(L, P_t) + (1-\delta)\mathsf{E}[V_{t+1}(P_{t+1}, M_{t+1})]. \tag{10}$$

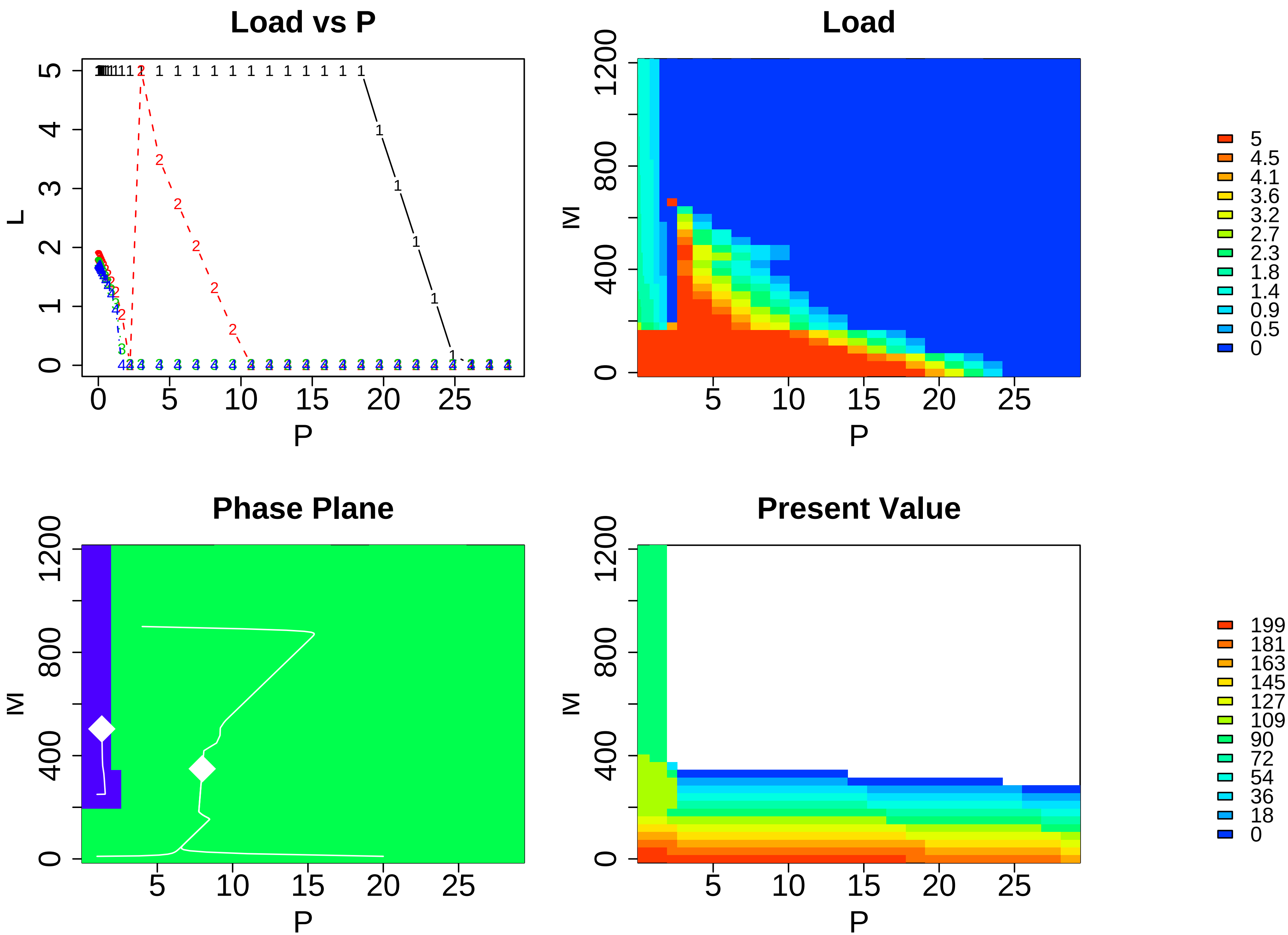
Here δ is the discount rate and $U(L,P_t)$ are the net benefits accruing in year t.

The optimal loading depends upon P_t and M_t. It is obtained by maximizing the right-hand side of (10).

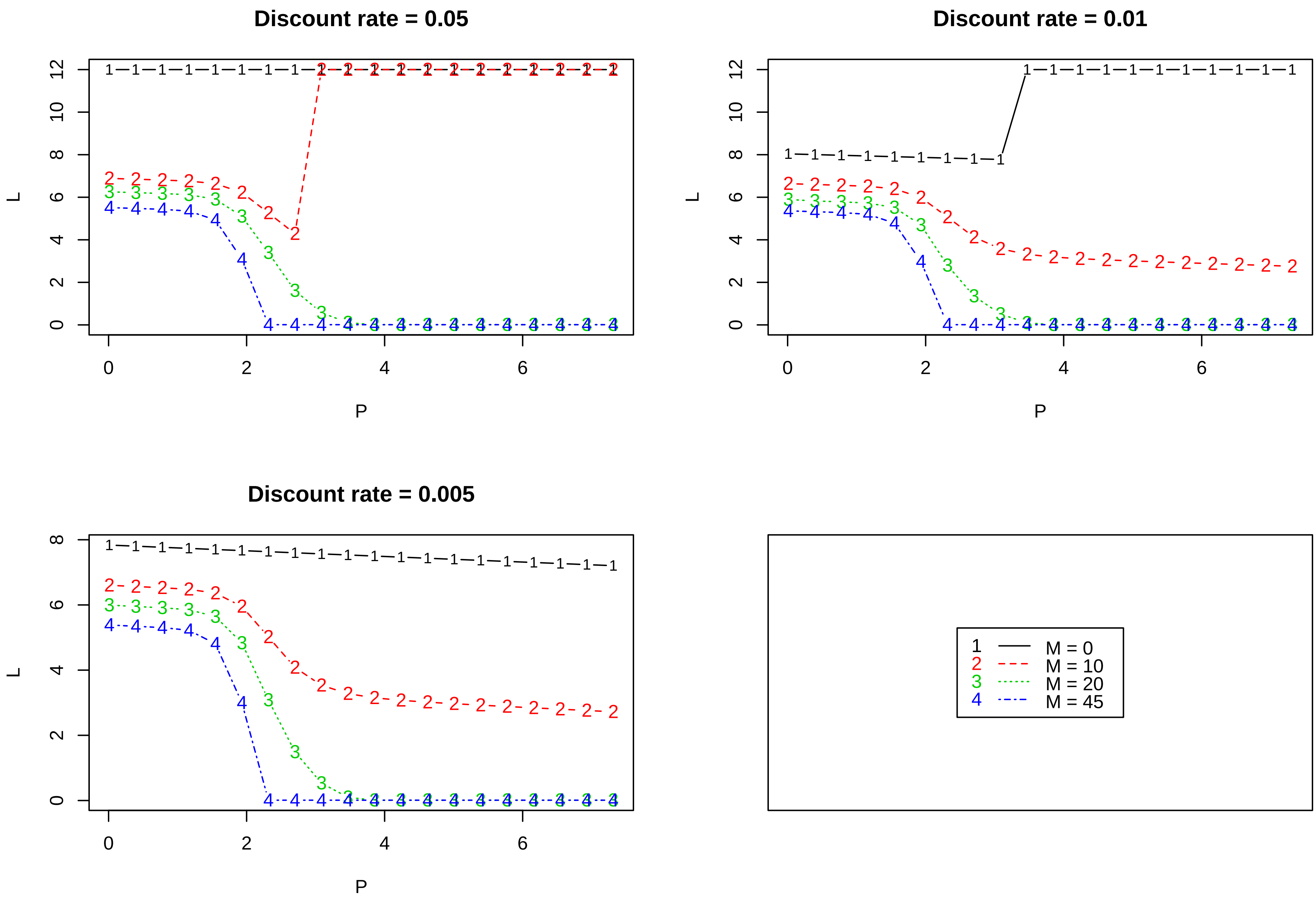
The equations are discretized in both state variables as well as the time, and maximization is performed by value and policy iterations.



Load vs P
L
P
Load
M
P
12
10.9
9.8
8.7
7.6
6.5
5.5
4.4
3.3
2.2
1.1
0
Phase plane
M
P
Present Value
M
P
93
85
76
68
59
51
42
34
25
17
8
0



Load vs P
L
P
Load
M
P
5
4.5
4.1
3.6
3.2
2.7
2.3
1.8
1.4
0.9
0.5
0
Phase Plane
M
P
Present Value
M
P
199
181
163
145
127
109
90
72
54
36
18
0


Discount rate = 0.05
Discount rate = 0.01
Discount rate = 0.005
L
P
1 M = 0
2 M = 10
3 M = 20
4 M = 45

Conclusions from the Lake Example

- A system may be stressed for a long period, may show few apparent effects of this stress, but suddenly may undergo a drastic, irreversible collapse.

- Uncertainty in the position of the threshold in recycling of phosphorus from sediments cannot be neglected for an optimal policy.

- Our calculations quantify a precautionary policy. The key is to monitor the slow variable and reduce loadings as resilience decreases.

- The precautionary policy takes advantage of the quasi-option value attached to P levels below the threshold for recycling.

Uncertainty in discount rate

Let the discount rate be a random variable X distributed as Gamma:

$$\Pr[x < X < x + dx] = \frac{\beta^\nu}{\Gamma(\nu)} x^{\nu-1} e^{-\beta x} dx. \tag{11}$$

Then the expected discount **factor** is given by

$$\begin{aligned} \mathsf{E}\left[e^{-Xt}\right] &= \frac{\beta^\nu}{\Gamma(\nu)} \int_0^\infty x^{\nu-1} e^{-(\beta+t)x} dx, \\ &= \left(1 + \frac{t}{\beta}\right)^{-\nu}. \end{aligned} \tag{12}$$

The corresponding discount **rate** is given by

$$\delta(t) = -\frac{d}{dt} \log \mathsf{E}\left[e^{-Xt}\right] = \frac{\nu}{\beta + t}. \tag{13}$$

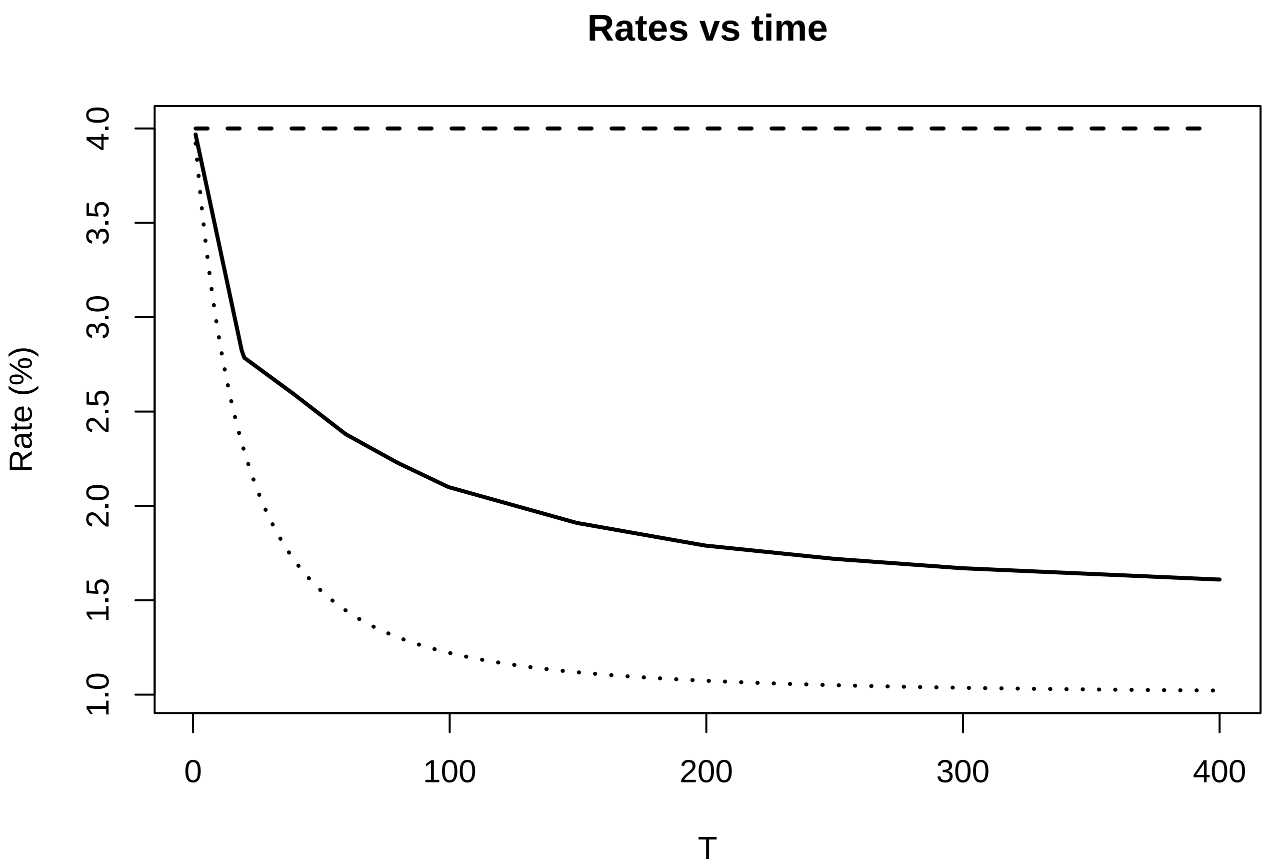
We have actually used rates that vary in time, motivated by a fit to long–term data from the US, and using a stochastic process for future discount rates.

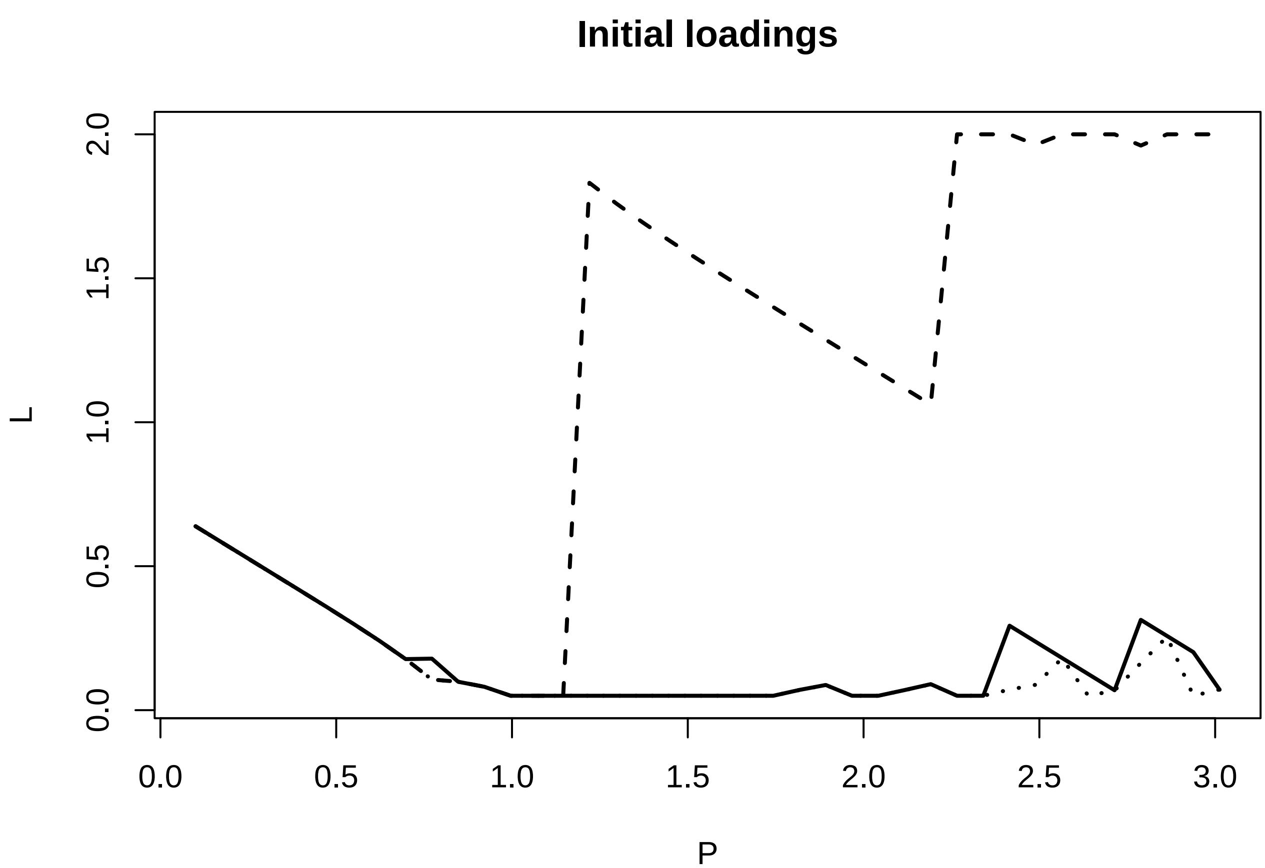
Three discounting schemes

1. Constant rate of 4% (dashed),

2. Best fit to US data (solid),

3. An estimated lower bound for future rates (dotted).


Rates vs time
Rate (%)
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0
100
200
300
400
T


Initial loadings
L
2.0
1.5
1.0
0.5
0.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
P

Conclusions

- Environmental decisions have consequences that span many generations.
- New economic theory provides support for policies that maintain ecosystem services over long time horizons, and prevent or mitigate environmental damage in the present.
- Such decisions are extremely sensitive to assumptions about discounting.
- At present, there is no single discounting scheme that dominates possible choices.
- We must scrutinize the choice of discounting scheme as carefully as any other modeling assumption.